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Comparative Study of Hexahedral and Tetrahedral Elements for Non-linear Structural Analysis

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1. INTRODUCTION

Finite elements are routinely used for analysis of real world problems in a wide range of engineering disciplines. The types of problems for which these are used include, but are not limited to, structural engineering, materials science, heat transfer, optics, and electromagnetics. While linearity is a good assumption to start with in many problems, reasonable solutions to real-life problems require them to be treated as non-linear. It is, therefore, necessary that the users of finite element codes be aware of the capabilities and limitations of their analysis tools.

One of the basic steps in modeling a problem with finite elements is to break up the computational domain into discrete elements. There exist automatic and semi-automatic mesh generation tools, which allow the analyst to mesh the problem domain with his or her choice of element shapes. Several element-level formulations are available in the literature for different element shapes. Hexahedral and tetrahedral element shapes are quite extensively employed. An earlier study on the relative performances of different element types in problems of linear structural analysis provides guidelines for the choice of hexahedral and tetrahedral elements. The present study attempts to extend the comparison to non-linear problems in structural analysis and provide similar guidelines.

2. NON-LINEAR BEHAVIOR

Non-linearities are encountered in almost all real-life engineering problems. These may be geometric, material or both. Geometric non-linearities arise because of large deflections and/or large rotations, even under small strains. The kinematic strain-displacement laws need to be modified suitably to incorporate geometric non-linearities. Material non-linearities, on the other hand, arise when the constitutive model itself is non-linear. These are incorporated through the stress-strain relations. For general use in stress analysis, finite elements must have a capability to handle non-linearities, both geometric as well as material. For this reason, it becomes imperative to test the response of the elements to both geometric and material non-linearities. A set of carefully chosen test problems is necessary for such a study. The test problems and the reasons for their choice are outlined in the next section.

3. TEST PROBLEMS

An element type used for general purpose non-linear finite analysis must be capable of representing large displacements, large rotations and large strains. While the first two problems test the geometric non-linear capabilities of the elements, the other problems test the behavior of the elements to represent non-linear material behavior accurately.

3. 1 Cantilever beam with concentrated load at free end

A cantilever beam loaded by a concentrated transverse load at the free end is considered. A schematic of the problem is shown in Figure 1. This problem is useful in testing the ability of the elements to withstand large deflections. Analytical solution is available [1] for the deflected shape of the beam, allowing the calculation of the errors associated with finite element analysis.

3. 2 Cantilever beam with concentrated moment at free end

A cantilever beam loaded by a concentrated moment at the free end is considered. A schematic of the problem is shown in Figure 2. The ability of the elements to accommodate large rotations as well as large displacements is verified in this problem.

3. 3 Hyperelastic cube under compression

A cube of hyperelastic material under compression is considered. The compression is applied as a displacement boundary condition on the top surface. Two cases of this problem are considered : (a) No friction exists at the top surface, and (b) an extreme case of friction is considered, where the top surface moves down rigidly. Schematics of the two cases are shown in Figures 3 and 4 respectively. In the absence of friction, the cube deforms to form a cuboid. When the top surface is moved down rigidly, there is a stress concentration at the corners and the material bulges out on all the sides. These problems test the ability of individual elements to reproduce large strains.

4. METHODOLOGY

The basic aim of this study is to compare the performance of tetrahedral and hexahedral elements in non-linear structural analysis. The comparison was made between three element types available in the commercial software ANSYS® : SOLID92, SOLID95R and SOLID45Ex (8 noded quadratic tetrahedron, 20 noded quadratic hexahedron with reduced integration, and 8 noded linear hexahedron with extra shapes, respectively). A "*best of class*" approach was used to select these three element types, based on some preliminary studies on linear structural analysis. For example, the 20 noded quadratic hexahedron with full integration (Solid95) was left out in favor of 20 noded quadratic element with reduced integration (Solid95R).

The aim in each test case is to measure a norm of computational effort (e.g., CPU time, NDOF, etc.) against a norm of computational performance (e.g., L_2 error norm for displacement, error in the energy norm, etc.) for each element type under consideration. The computational effort is varied by using different meshes.

5. RESULTS AND DISCUSSION

Non-linear problems are generally solved by linearization. In the context of finite element analysis, the linearization is embedded in the formulation most of the times. An implication of this is that there are additional set of iterations that need to be performed to obtain a numerical solution. Given a finite element mesh, the accuracy of the solution as well as the computational effort needed to obtain it, are dependent on how a non-linear load-step is linearized. If the load step is linearized with few increments, unconverged equilibrium iterations may inflate the total CPU time consumed for solving a problem. Therefore, in non-linear analysis, reporting CPU time as a measure of computational effort is not completely satisfactory. With this in mind, all the data for the present study is presented in terms of number of degrees of freedom (NDOF), with an understanding that each degree of freedom takes approximately same amount of CPU time for solution, when a proper linearization of the load is used.

Figures 5 and 6 show the L2 norm of error in displacement and error in strain energy, for the case of bending of a cantilever beam under concentrated load. The linear hexahedral element with extra shapes (45E) performs the best, followed by quadratic hexahedral element with reduced integration (95R) and the quadratic tetrahedral element (92). Figures 7 and 8 show similar trends for a higher non-linear load. It is seen in these cases that the error does not decrease in proportion to the increase in NDOF. This may be because of a stiff response (locking) of the elements to bending loads. If, indeed, such is the case, then predominant part of the error in solution is because of the stiff response. In that case, no strong conclusions can be drawn from this data. This needs to be investigated in further detail.

Figures 9 and 10 show the L2 norm of error in displacement and error in strain energy, for the case of bending of a cantilever beam under concentrated moment. Again, the linear hexahedral element with extra shapes (45E) performs the best, followed by quadratic hexahedral element with reduced integration (95R) and the quadratic tetrahedral element (92). Figures 11 and 12 show similar trends for a higher value of moment. The behavior in these cases is similar to that in the case of bending under a point load. The observations made about locking are true for this test problem too.

The problem of compression of a hyperelastic cube tests the large strain capabilities of the elements thoroughly. This problem shows a slightly different trend for the relative performance of different element types. Figures 13, 14 and 15 show the performance trends. The element type 95R is the best, followed by 92 and the linear element 45E. The results are consistent with and without friction during compression. This set of results is slightly different from the cases of bending. This makes it even more important to carefully investigate whether the errors in the bending analyses are predominantly due to locking.

REFERENCES

1. Frisch-Fay, R., "A New Approach to the Analysis of the Deflection of Thin Cantilevers", *Journal of Applied Mechanics*, **28**, 87-90, 1961.

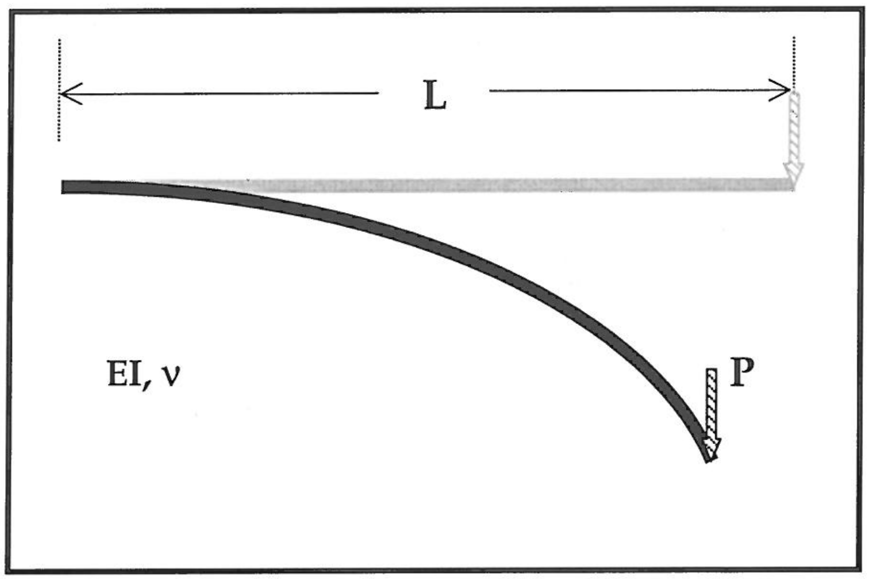

Figure 1. Cantilever beam under transverse load

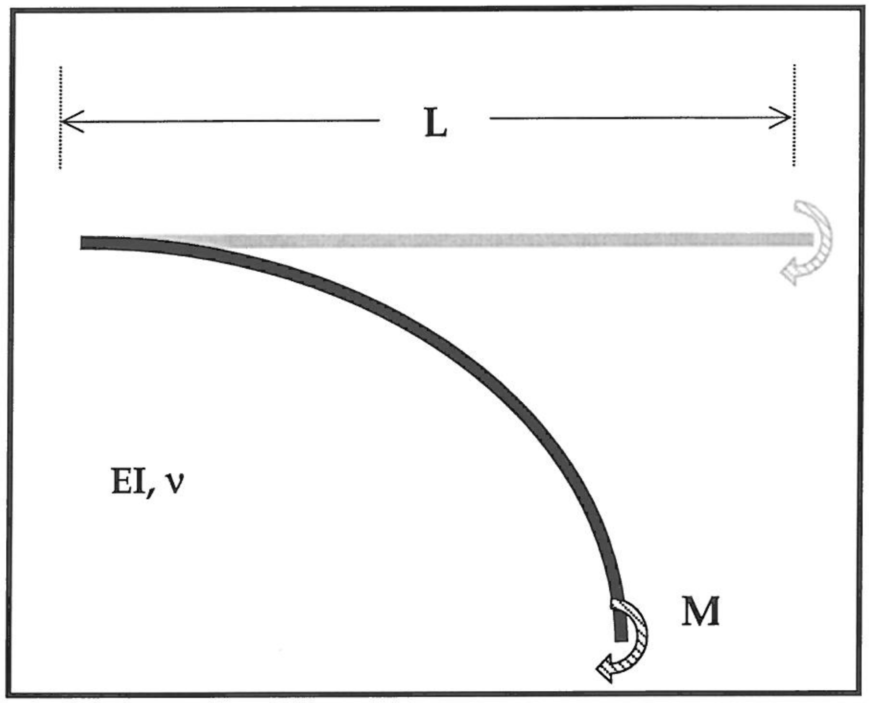

Figure 2. Cantilever beam under concentrated moment

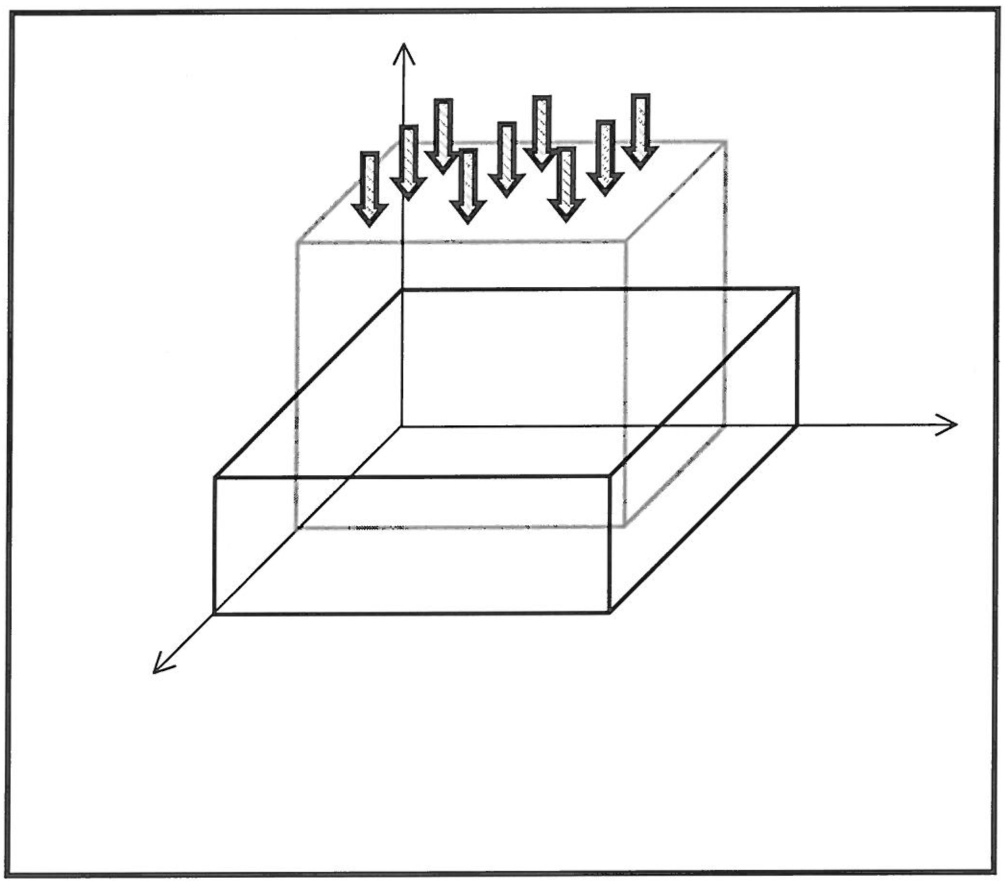

Figure 3. Hyperelastic cube under compression without friction on the top surface

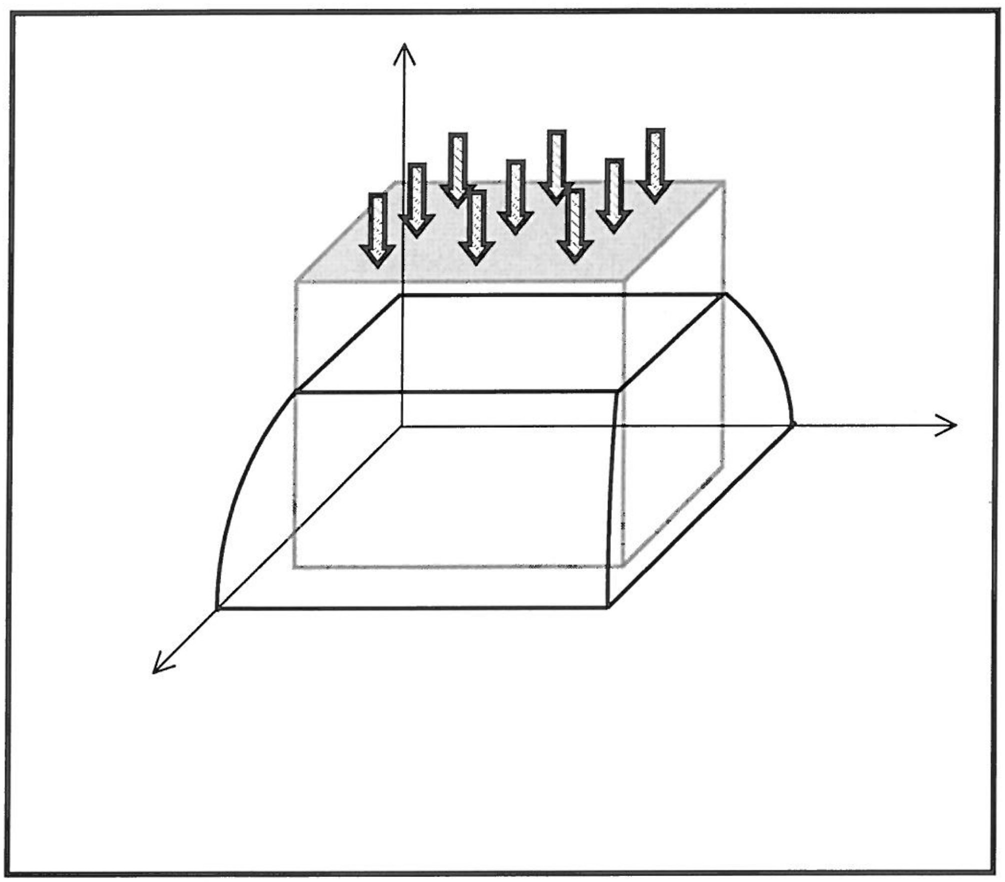

Figure 4. Hyperelastic cube under compression with friction on the top surface

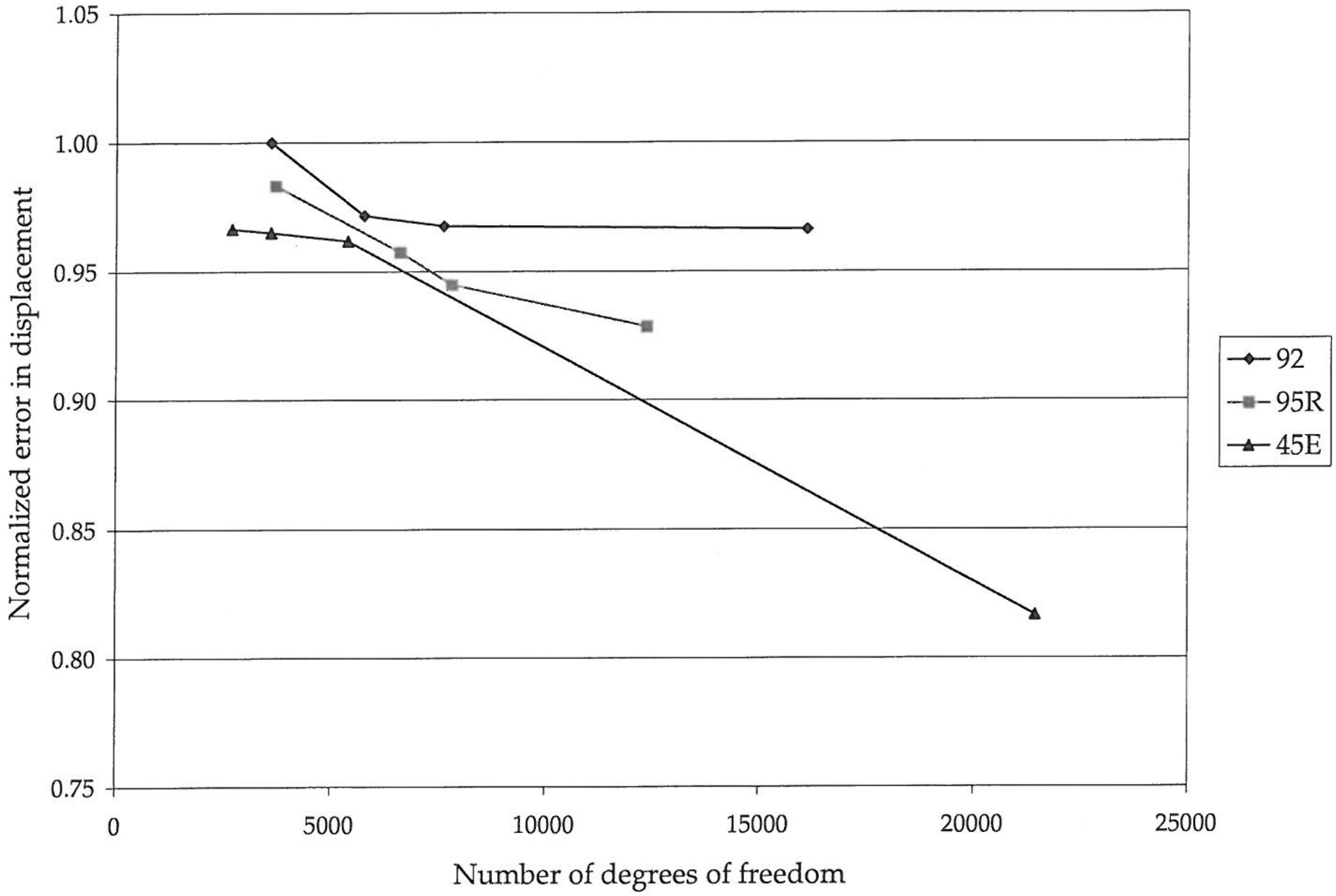

Figure 5. L2 error norm of displacement vs. NDOF for cantilever beam under concentrated load, $PL^2/EI=6$

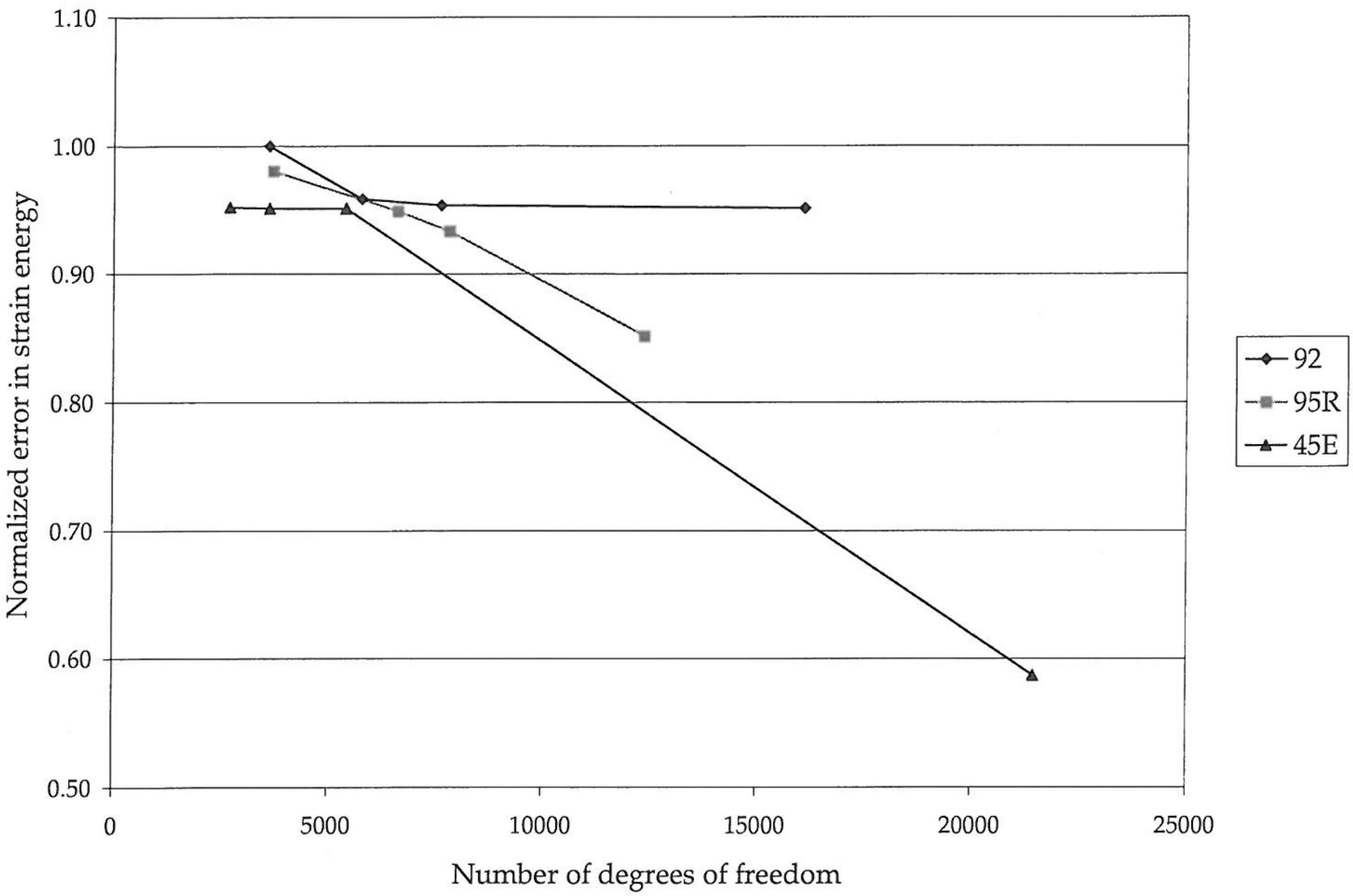

Figure 6. Error in strain energy vs. NDOF for cantilever beam under concentrated load, $PL^2/EI=6$

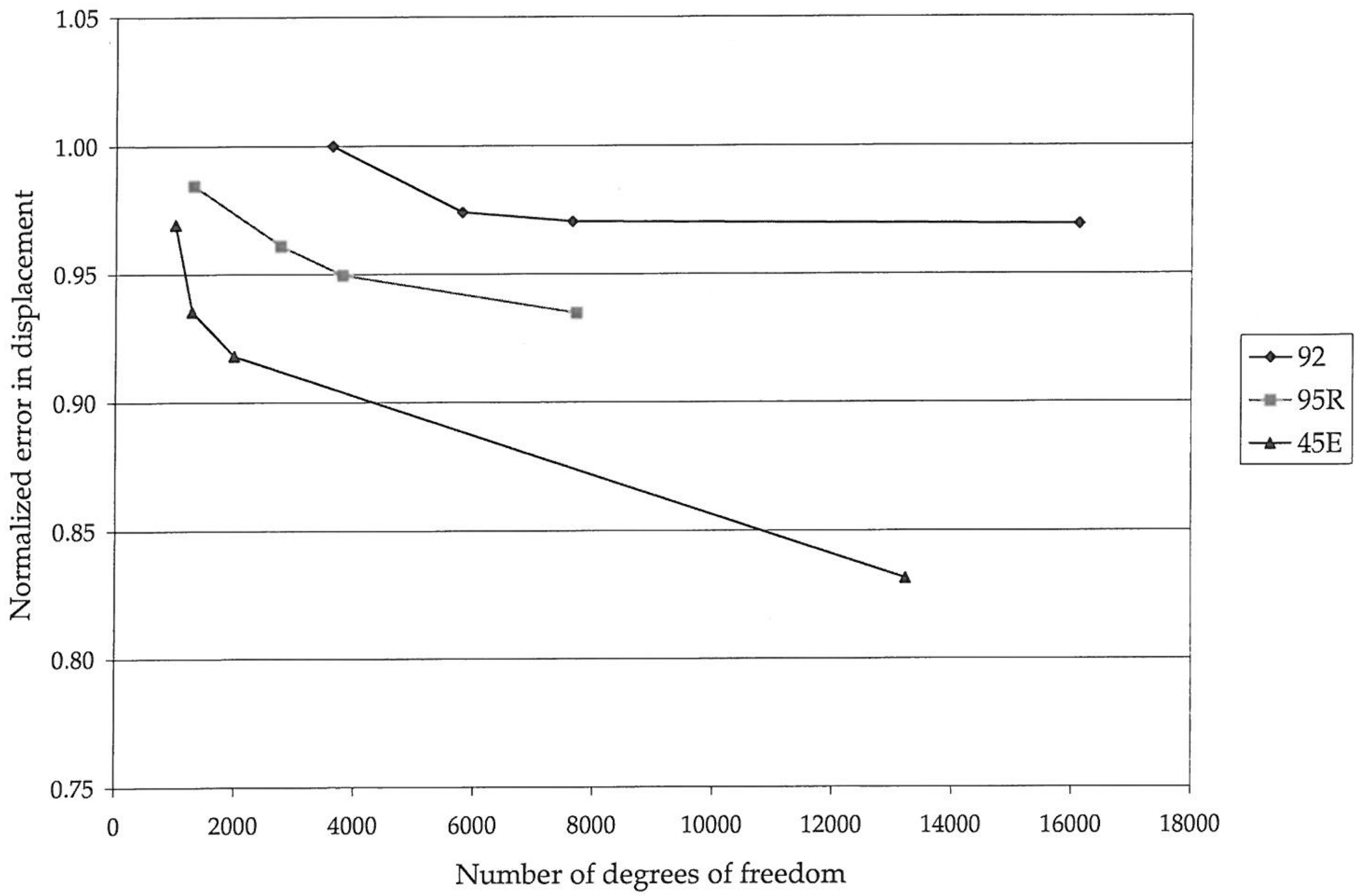

Figure 7. L2 error norm of displacement vs. NDOF for cantilever beam under concentrated load, $PL^2/EI=12$

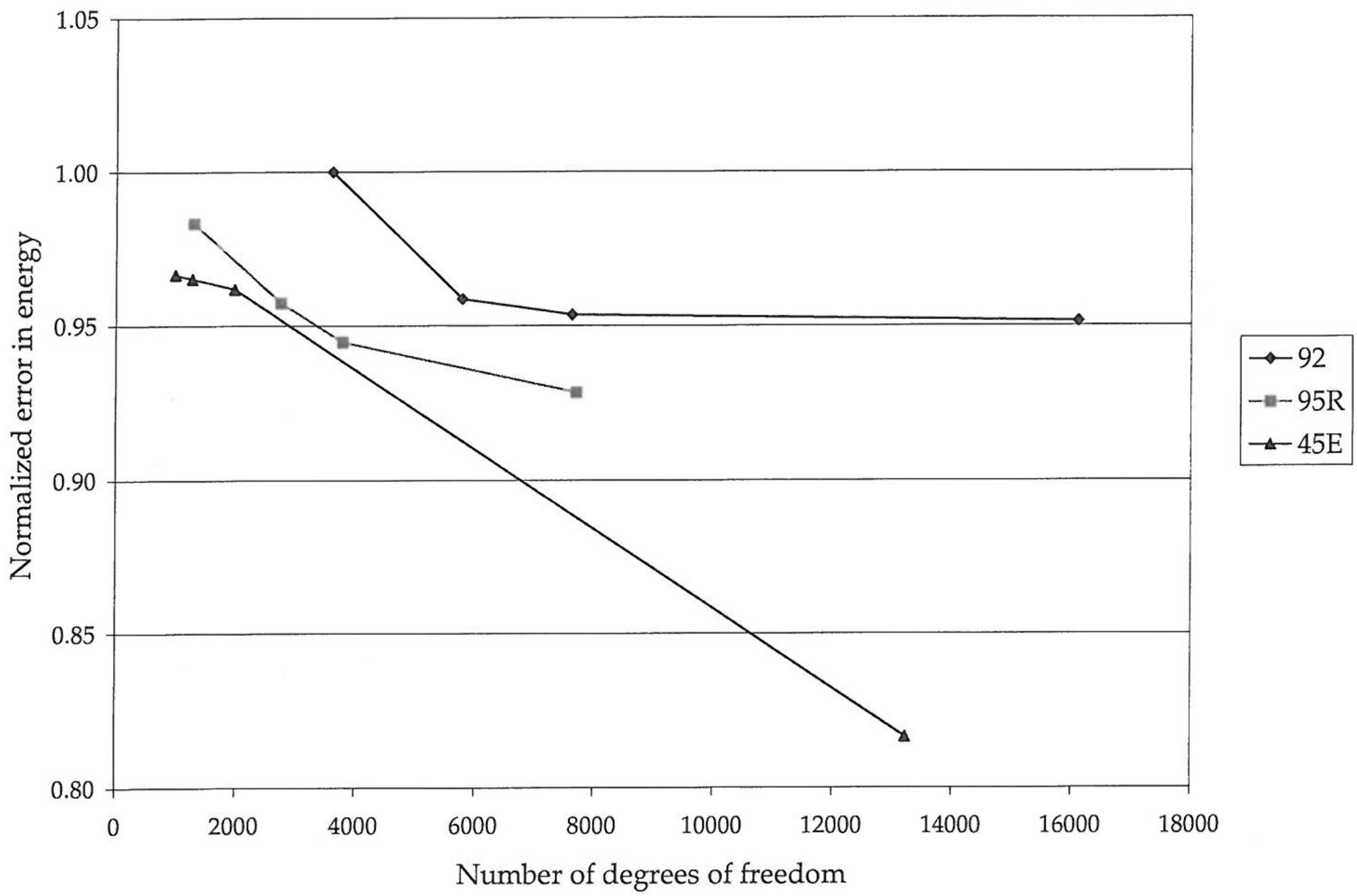

Figure 8. Error in strain energy vs. NDOF
for cantilever beam under concentrated load, $PL^2/EI=12$

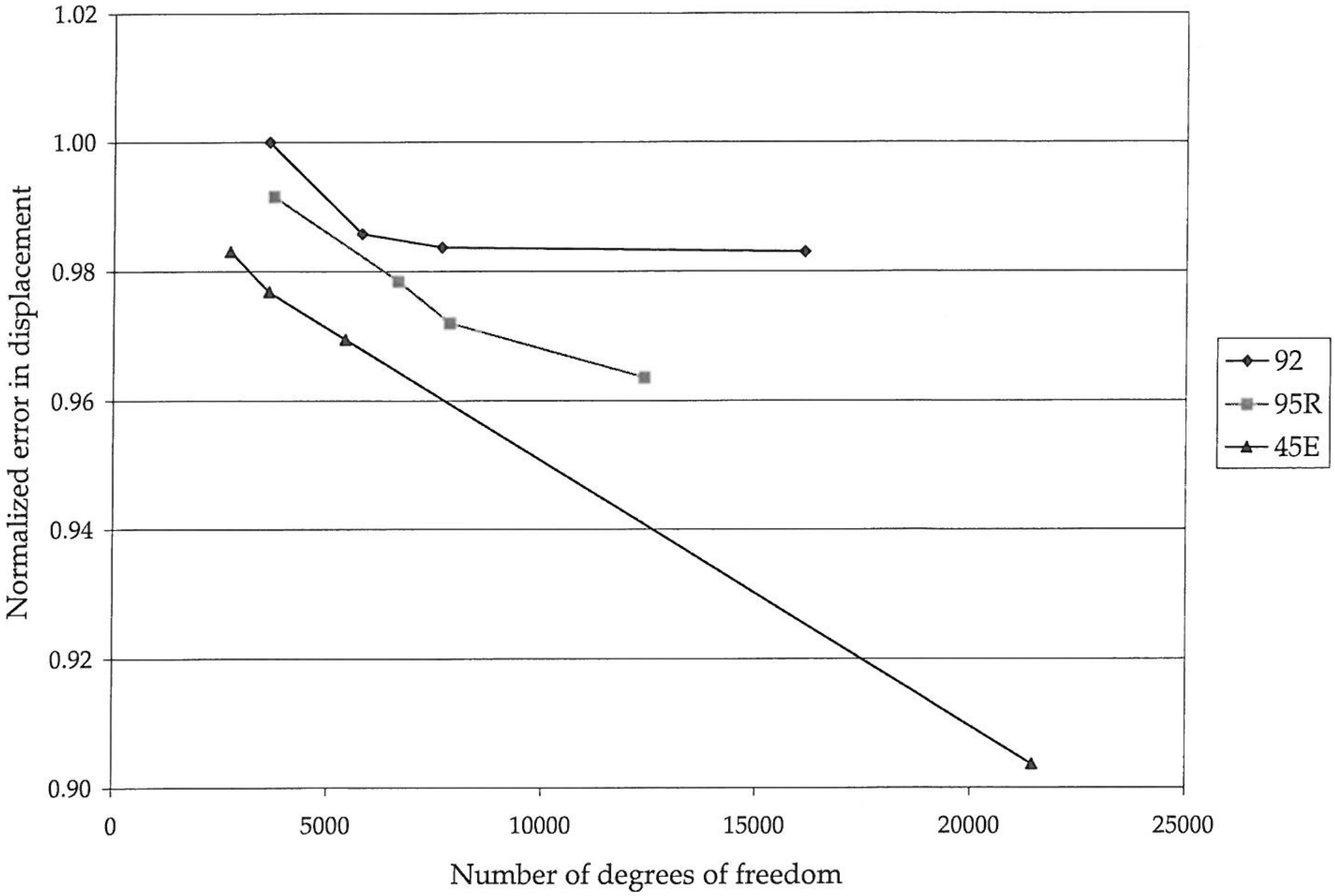

Figure 9. L2 error norm of displacement vs. NDOF for cantilever beam under concentrated moment, ML/EI=6

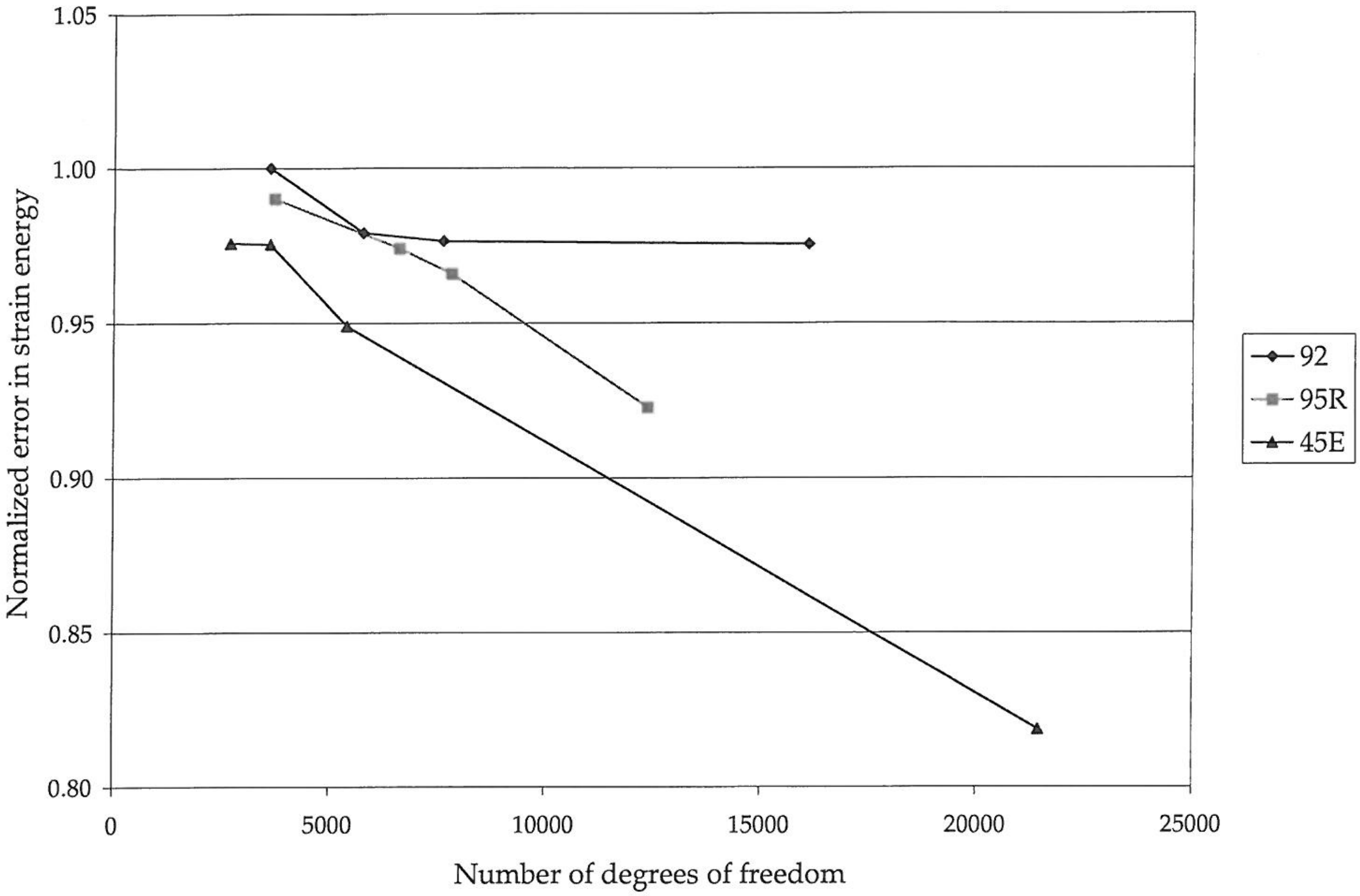

Figure 10. Error in strain energy vs. NDOF for cantilever beam under concentrated moment, ML/EI=6

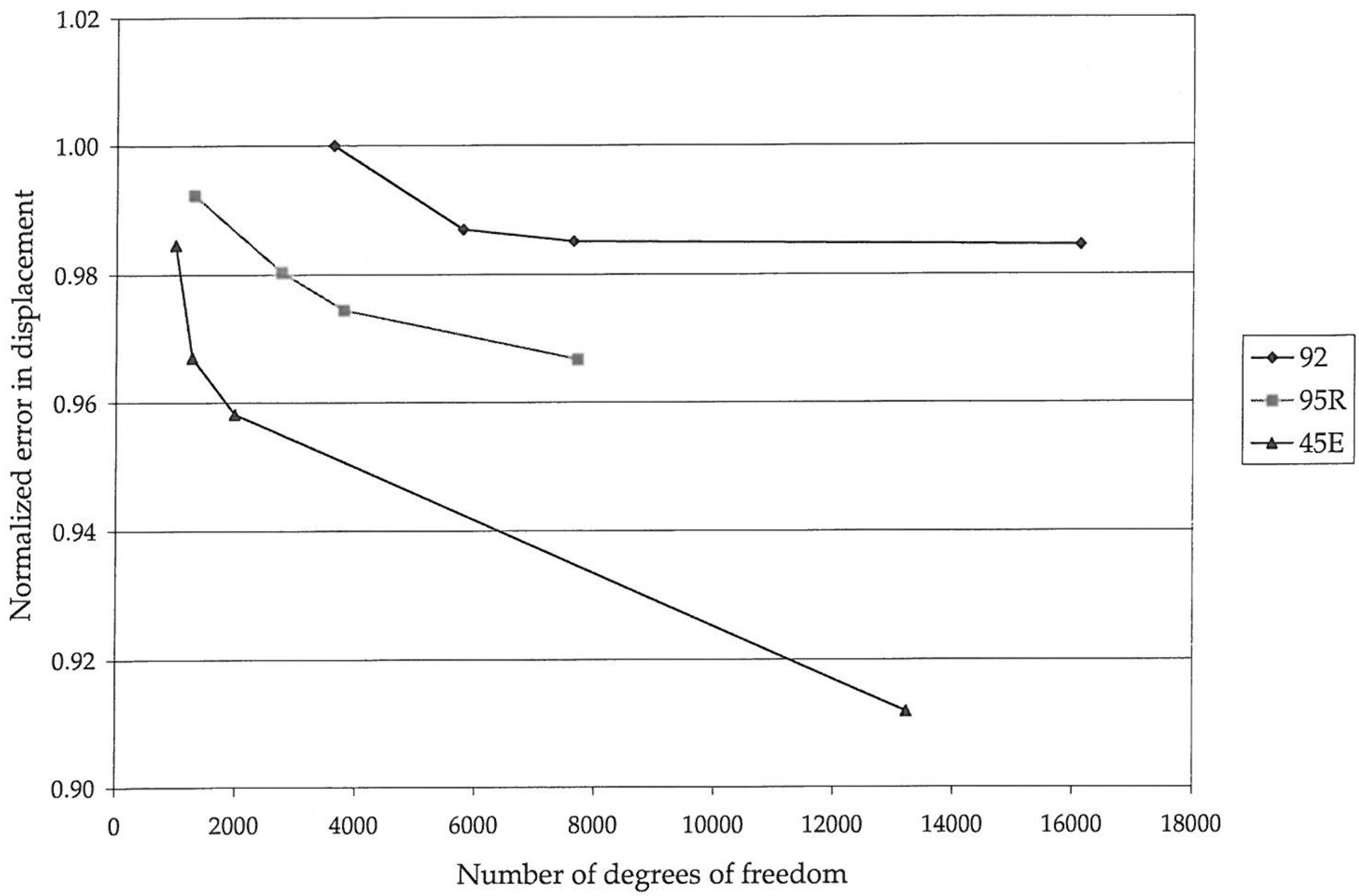

Figure 11. L2 error norm of displacement vs. NDOF for cantilever beam under concentrated moment, ML/EI=12

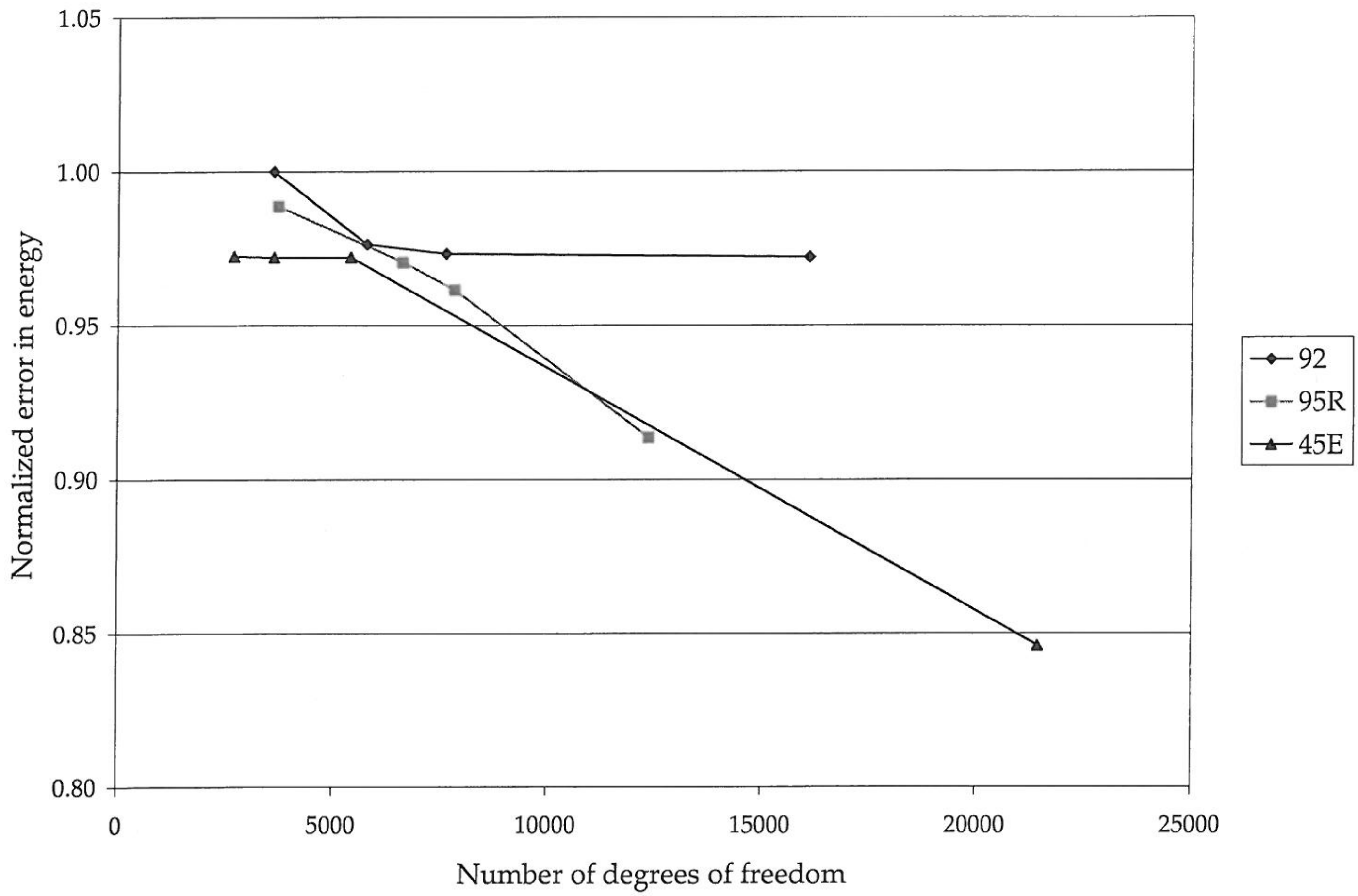

Figure 12. Error in strain energy vs. NDOF for cantilever beam under concentrated moment, ML/EI=12

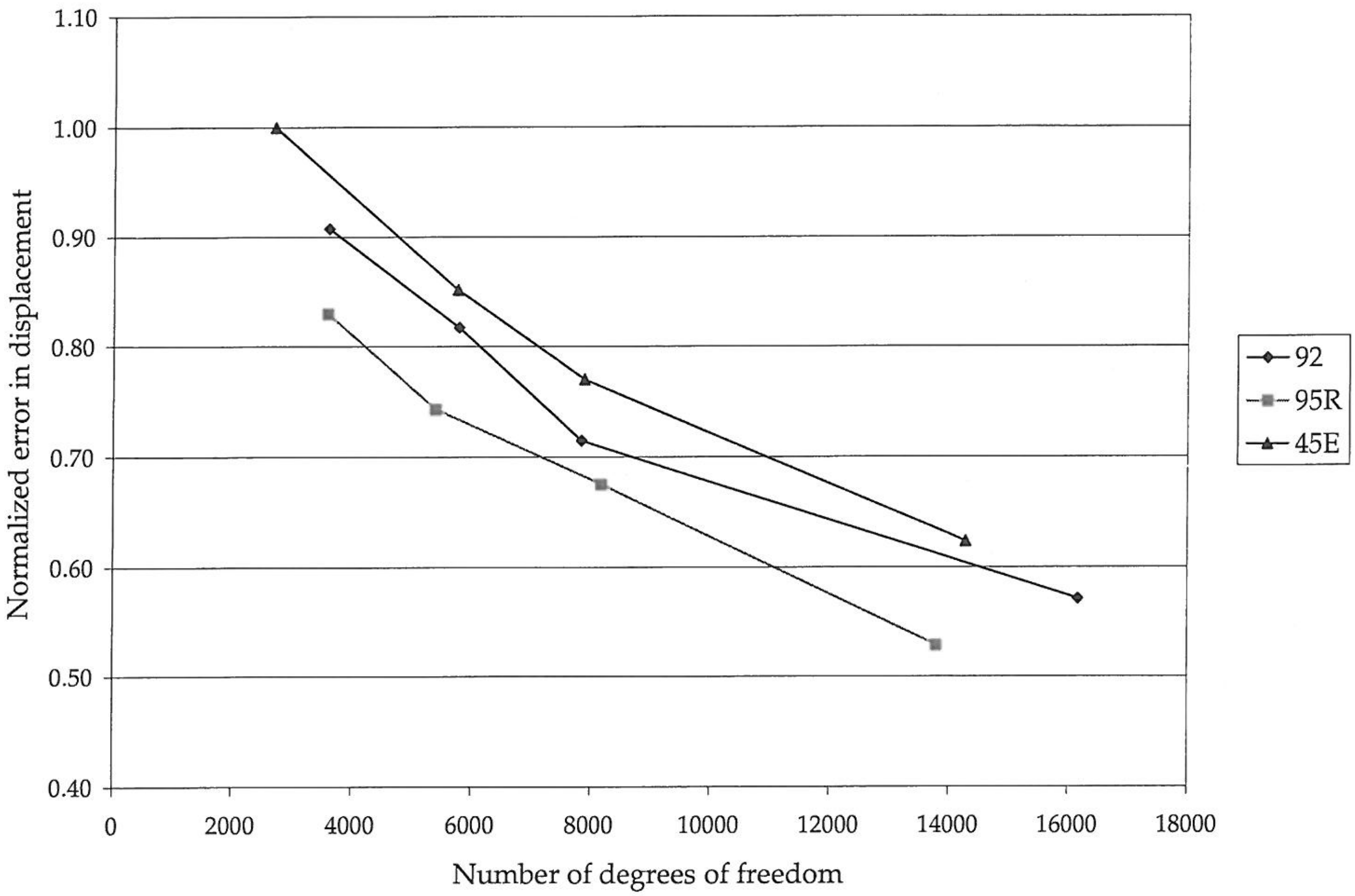

Figure 13. L2 error norm of displacement vs. NDOF for hyperelastic cube under compression without friction

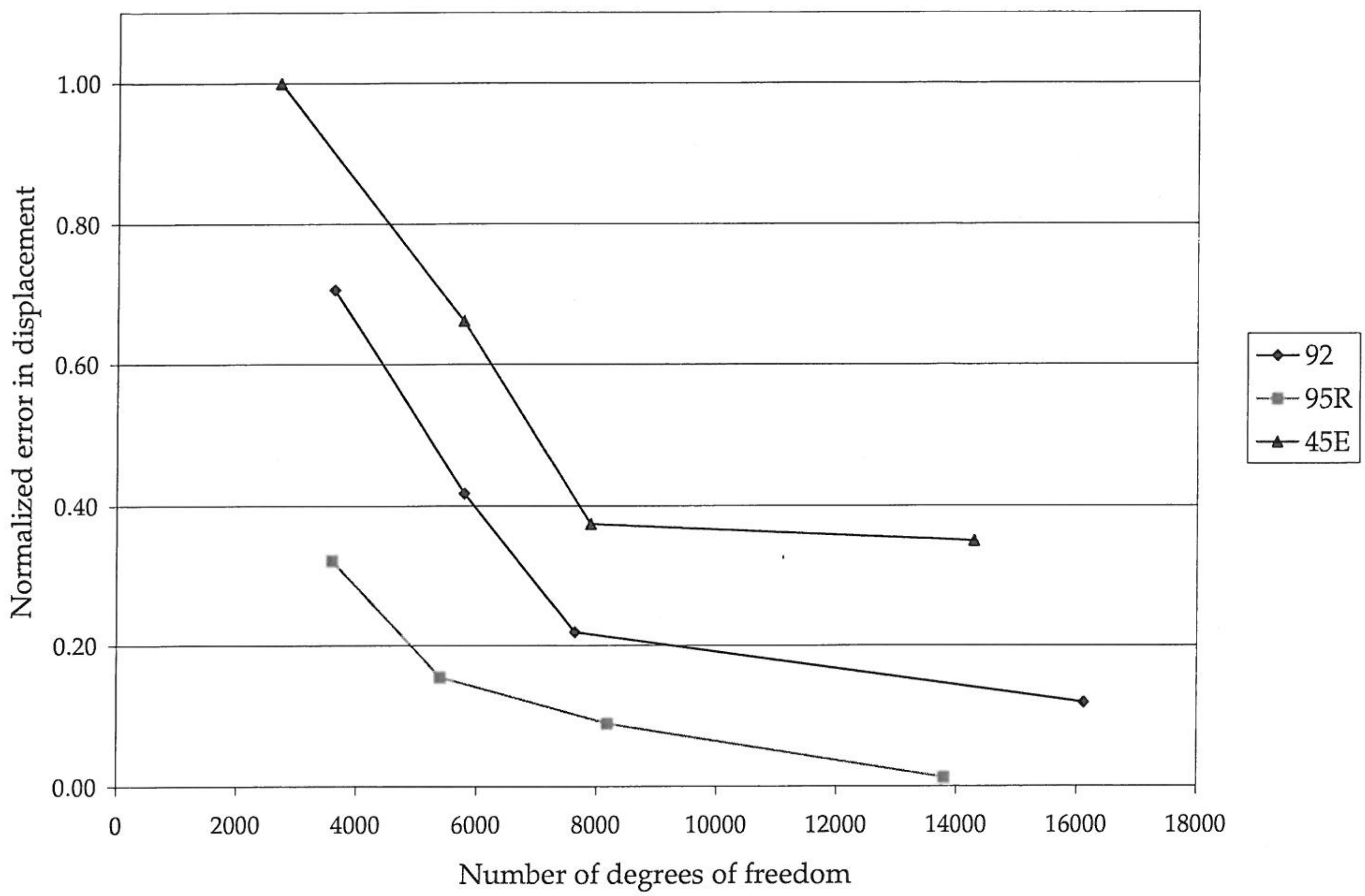

Figure 14. Error in strain energy vs. NDOF for hyperelastic cube under compression without friction

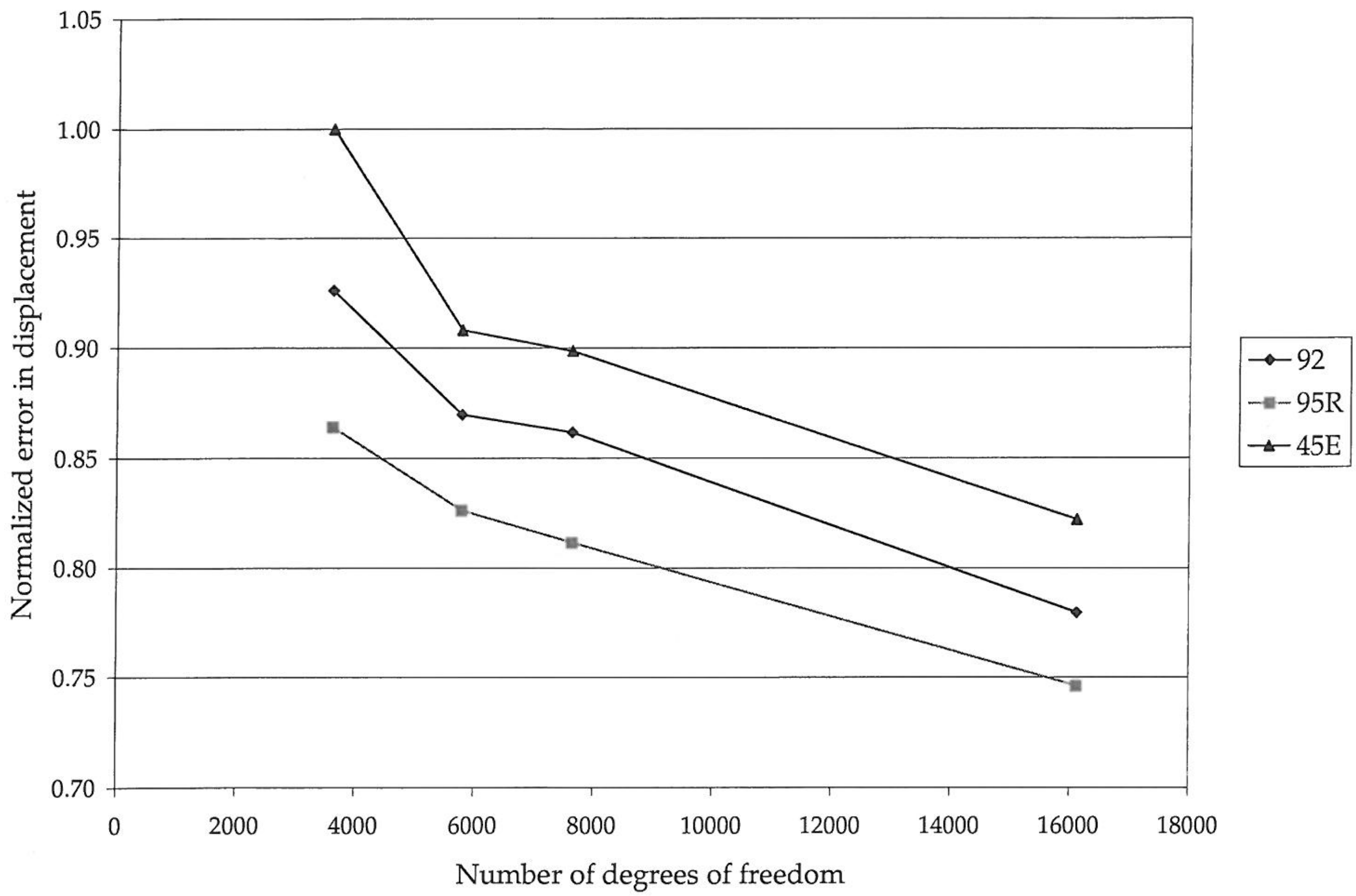

Figure 15. L2 error norm of displacement vs. NDOF for hyperelastic cube under compression with friction